

# Entropy and the Cosmic Ray Particle Energy Distribution Power Law Exponent

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We consider the hypothesis that cosmic rays are emitted from the surfaces of neutron stars by a process of evaporation from an internal nuclear liquid to a dilute external gas which constitutes the “vacuum”. On this basis, we find an inverse power in the energy distribution with a power law exponent of 2.701178 in excellent agreement with the experimental value of 2.7. The heat of nuclear matter evaporation via the entropy allows for the computation of the exponent. The evaporation model employed is based on the entropy considerations of Landau and Fermi that have been applied to the liquid drop model of evaporation in a heavy nucleus excited by a collision. This model provides a new means of obtaining power law distributions for cosmic ray energy distributions and, remarkably, an actual value for the exponent which is in agreement with experiment and explains the otherwise puzzling smoothness of the cosmic ray energy distribution over a wide range of energies without discontinuities due to contributions from different sources required by current models.

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## I. INTRODUCTION

The classic treatises on sources of cosmic ray energy distributions[1–3] discuss a power law probability rule whose theoretical basis is presently not entirely clear[4]. A variety of mechanisms have been proposed, assuming either top-down, where initially very high energy particles come from decays of heavy remnants from the early universe and bottom-up ones which involve cosmic accelerators of various kinds, including “one-shot” acceleration, perhaps around neutron stars or black holes where large electric fields might be found with relatively small orthogonal magnetic fields which would otherwise produce energy losses due to synchrotron radiation, and diffusive (Fermi) acceleration via collisions of charged particles with moving magnetic fields due to astrophysical shock waves.

Confining attention to the region in which the observed differential flux with respect to energy  $dN/dE$  is proportional to  $E^{-\alpha}$  with  $\alpha$  near 2.7, there are at least two notable problems with traditional explanations. The first is obtaining the exponent 2.7. While Fermi acceleration can at least give rise to a power law spectrum, it is difficult to argue for the correct exponent. In first order Fermi acceleration the exponent is 1 plus a source-dependent correction, while second order Fermi acceleration gives an (incorrect) exponent of 2. A short and very readable account of high energy cosmic ray acceleration mechanisms can be found in [5].

Secondly, since multiple sources would contribute to what is measured on earth, one has to explain: 1) why the observed spectrum is as smooth as it is (*i.e.* why each source has, within observed errors, the same exponent), and 2) why various sources should be distributed in such a way and with such intensities that there would, even with the same slopes, be no jumps in the spectrum.

This suggests that a more economical explanation not

dependent on such apparent fine-tuning, and, if possible, giving the correct exponent, would be worth considering.

A statistical thermodynamic view of the energy distribution was pioneered employing a non-extensive entropy[6–9] and later interpreted in terms of temperature fluctuations[10–13]. For our purposes we apply *entropy computations* in a form originally due to Landau[14] for the Landau-Fermi liquid drop model of a heavy nucleus. The energy distribution of some of the decay products are thought to be evaporating nucleons from the bulk liquid drops excited by a heavy nuclear collision[15]. Although the entropy for this model is computed from non-relativistic quantum mechanics, the cosmic ray version of evaporation of course requires an ultra-relativistic limit.

Our purpose is to explain the empirical measured scaling law index  $\alpha$  which appears in the energy distribution law of cosmic ray nuclei via the differential flux

$$\left[ \frac{d^4 \bar{N}}{dt dA d\Omega dE} \right] \approx \frac{1.8 \text{ Nucleons}}{\text{sec cm}^2 \text{ sr GeV}} \left( \frac{1 \text{ GeV}}{E} \right)^\alpha \quad (1)$$

that holds experimentally true[16] for the energy interval  $5 \text{ GeV} < E < 100 \text{ TeV}$ . Our theoretical result is that

$$\alpha = 3 \left[ \frac{\zeta(4)}{\zeta(3)} \right] \quad (2)$$

wherein

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (3)$$

is the Riemann zeta function. Numerically, our theoretical value is

$$\alpha_{\text{th}} \approx 2.701178 \quad (\text{theory}) \quad (4)$$

in *complete agreement* with the value  $\alpha_{\text{exp}} \approx 2.7$  measured in cosmic ray experiments. The theory is based on

computing the entropy per particle emitted from a cosmic ray source to be described more fully in what follows.

In Sec.II we consider the evaporation of nucleons from a low temperature Landau-Fermi liquid droplet excited by an external collision. If one knows the entropy per nucleon  $\Delta s$  in the excited nuclear state, then the heat of evaporation  $q_{\text{vaporization}} = T\Delta s$  determines the energy distribution of vaporized nucleons employing the activation probability

$$P(E) = \exp(-\Delta s(E)/k_B). \quad (5)$$

The quasi-particle entropy for a non-relativistic nucleon liquid droplet is reviewed below.

In Sec.III, we propose as the *sources* of cosmic rays, *the evaporating stellar winds from neutron star surfaces*. The quantum hadronic dynamical models of nuclear liquids have been a central theoretical feature of nuclear matter[17–19]. The theory of quantum hadronic matter is modeled in the main as an effective Bose (collective meson) theory. These models involve condensed scalar and vector mesons in about equal amounts. The scalar field may be thought to describe collective alpha nuclei embedded in the liquid while the vector field may be thought to describe deuteron nuclei embedded in the liquid. The neutron stars are evaporating through the surface from the nuclear matter within the bulk liquid into the “vacuum” or dilute gas. The gas contains the stellar wind of cosmic rays emanating from the neutron star sources. In Sec.IV, the exponent in Eqs.(2) and (4) are computed. In the concluding Sec.V the physics of the model is further discussed.

## II. EVAPORATION OF FLUID PARTICLES

The heat capacity per nucleon in a non-relativistic low temperature  $T$  Landau-Fermi liquid drop is given by

$$c = \frac{dE}{dT} = T \frac{ds}{dT} = \gamma T \quad \text{as } T \rightarrow 0. \quad (6)$$

Eq.(6) implies an excitation energy  $E = (\gamma/2)T^2$  and an entropy  $\Delta s = \gamma T$  so that

$$\Delta s(E) = \sqrt{2\gamma E} = k_B \sqrt{\frac{E}{E_0}}. \quad (7)$$

When a large nuclear matter nucleus is excited by an external collision or otherwise caused to be at a nonzero temperature the energy distribution of nucleons that are evaporated is given by Eqs.(5) and (7) imply

$$P(E) = \exp\left(-\sqrt{\frac{E}{E_0}}\right). \quad (8)$$

Let us now consider cosmic energy sources.

## III. COSMIC RAY EVAPORATION

The structure of neutron stars consists of a big nuclear droplet[20, 21] facing a very dilute gas, *i.e.* the “vacuum”. Neutron stars differ from being simply very large nuclei in that most of their binding is gravitational rather than nuclear, but, the droplet model of nuclear model should still offer a good description of nuclear matter near the surface where it can evaporate.

The evaporation is from the effective fields in the form of scalar nuclei  ${}^4\text{He} \equiv \alpha$  and vector deuterons  ${}^2\text{H} \equiv d$ . Note that from neutron stars we will not be in the low temperature regime discussed in the previous section, but rather at high temperatures where the heat capacity goes to a constant.

What evaporates from the neutron star via scalar and vector fields are then energetic protons and alpha particles along with other nuclei to a much lesser extent. Deuterons are only weakly bound and would be expected to photodissociate on background photons present throughout space rapidly to produce protons and neutrons which in turn would produce protons when they decay. In any event, experimentally, of course, at high energies discrimination of light nuclei and protons is experimentally difficult. The energy distribution used in the argument that follows comes directly from the entropy of scalar (spin zero) and vector (spin 1) combinations of baryons.

## IV. THE POWER LAW EXPONENT

Ultra relativistic particles are virtually massless and thus have a density of energy states  $\propto \epsilon^2$ . The mean energy per particle then obeys

$$E = \frac{\int_0^\infty f(\epsilon) \epsilon^3 d\epsilon}{\int_0^\infty f(\epsilon) \epsilon^2 d\epsilon} \quad (9)$$

wherein the Bose distribution is

$$f(\epsilon) = \frac{1}{e^{\epsilon/k_B T} - 1}. \quad (10)$$

Employing the Gamma and zeta functions

$$\begin{aligned} \Gamma(s) &= \int_0^\infty e^{-x} x^s \left[ \frac{dx}{x} \right], \\ \Gamma(s)\zeta(s) &= \int_0^\infty \left[ \frac{x^s}{e^x - 1} \right] \left[ \frac{dx}{x} \right], \end{aligned} \quad (11)$$

yields the mean energy

$$E = cT \quad \text{wherein} \quad c = k_B \left[ \frac{\Gamma(4)\zeta(4)}{\Gamma(3)\zeta(3)} \right]. \quad (12)$$

The heat capacity per evaporated boson is thereby

$$c = \alpha k_B = T \frac{ds}{dT} = \frac{dE}{dT} \quad (13)$$

wherein  $\alpha$  is given in Eq.(2). From Eqs.(12) and (13), the entropy obeys

$$s(E) = \alpha k_B \ln \left( \frac{E}{E_0} \right). \quad (14)$$

The power law energy distribution follows from Eqs.(14) and (5),

$$P(E) \propto \left( \frac{E_0}{E} \right)^\alpha \quad (15)$$

which is the central result of this work.

## V. CONCLUSIONS

We have explored the hypothesis that cosmic rays are emitted from the surfaces of neutron stars. The cosmic rays themselves are in the stellar atmospheric winds

blowing away from the neutron star source. The cosmic rays are equivalently nuclei which are evaporating from the bulk of the neutron star. On this basis, we find an inverse power in the energy distribution with a power law exponent of  $\alpha = 2.701178$  in excellent agreement with experimental data. The method of computing the energy distribution of the evaporated cosmic rays is closely analogous to those employed by Landau and Fermi for evaporation of nucleons from the Bohr-Mottelson liquid drop model. The correct exponent predicted by the model is satisfactory and encouraging. Further work is in progress with more refined models.

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- [1] T. K. Gaisser, “*Cosmic Rays and Particle Physics*”, Cambridge University Press, Cambridge (1990).
  - [2] T. Stanev, “*High Energy Cosmic Rays*”, Springer, (2004).
  - [3] A. Alexander *et al.* the Pierre Auger Collaboration, The Pierre Auger Observatory, Contributions to the 33rd International Cosmic Ray Conference (ICRC 2013).
  - [4] T. K. Gaisser, T. Stanev, Cosmic Rays, in K. Hagiwara *et al.*, *Phys. Rev. D* **66**, 010001-1 (2002).
  - [5] M. Bustamante *et al.*, “*High Energy Cosmic Ray Acceleration*”, in Proceedings of the 5th CERN-Latin American School of High-Energy Physics, Recinto Quirama (Medellin), Antioquia, Colombia, 15 - 28 Mar 2009, pp.533-540 DOI 10.5170/CERN-2010-001.533
  - [6] C. Tsallis, *J. Stat. Phys.* **52**, 479 (1988).
  - [7] C. Tsallis, R. S. Mendes and A. R. Plastino, *Physica*, **261A**, 534 (1998).
  - [8] C. Tsallis, J. C. Anjos, E. P. Borges, astro-ph/0203258 (2004).
  - [9] G. Wilk and Z. Włodarczyk, *Phys. Rev. Lett.* **84**, 2770 (2000).
  - [10] C. Beck, *Phys. Rev. Lett.* **87**, 180601 (2001).
  - [11] C. Beck and E. G. D. Cohen, *Physica A* **322**, 267 (2003).
  - [12] C. Beck and E. G. D. Cohen, *Physica A* **344**, 393 (2004).
  - [13] C. Beck, E. G. D. Cohen, and H. L. Swinney, *Phys. Rev. E* **72**, 026304 (2005).
  - [14] L. D. Landau and Ya. Smorodinsky, “*Lectures on Nuclear Theory*”, Lecture Seven, Plenum Press, New York (1959).
  - [15] J. M. Blatt and V. E. Weisskopf, “*Theoretical Nuclear Physics*”, Chapt. VIII, Sec. 6, Springer-Verlag, New York (1979).
  - [16] D. E. Groom *et al.*, *European Physical Journal C* **15**, 1 (2000).
  - [17] John D. Walecka. “*Theoretical Nuclear and Subnuclear Physics*”, Chapters 6 and 25, Oxford University Press, Oxford (1996).
  - [18] B. D. Serot and J. D. Walecka, *Adv. Nuc. Phys.* **16**, 1 (1986).
  - [19] L. Scalone, M. Colonna and M. Di Toro, *Phys. Lett. B* **461**, 9 (1999).
  - [20] P. Haensel, A. T. Potekhin, D. G. Yakovlev, “*Neutron Stars I-Equation of State and Structure*”, Springer Verlag Berlin(2007).
  - [21] Meng Jin, Ji-Sheng Chen and Jia-Rong Li, *Commun. Theor. Phys.* **40**, 715 (2003).